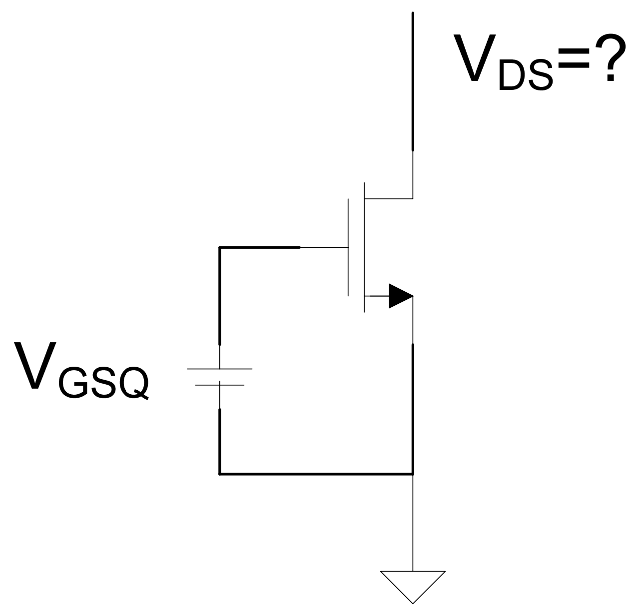
EE 434
Lecture 17

Small Signal Models

Quiz 11

What is the minimum voltage at the drain of the MOS transistor that will cause the inversion layer at the interface between the drain and source to disappear? Assume the source is connected to ground and a dc voltage source of value V_{GSQ} is connected to the gate.



And the number is ….

1 8 7 5 3

6 9 4 2

And the number is ….

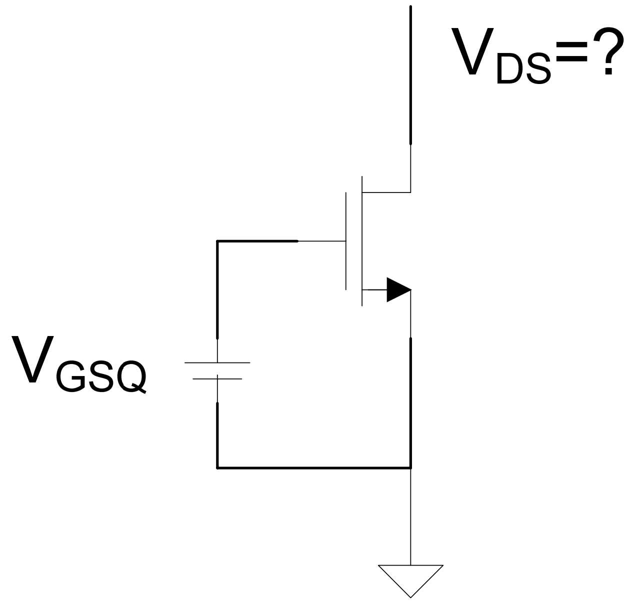
1 8 7 5 3

6 9 4 2

1

Quiz 11 Solution:

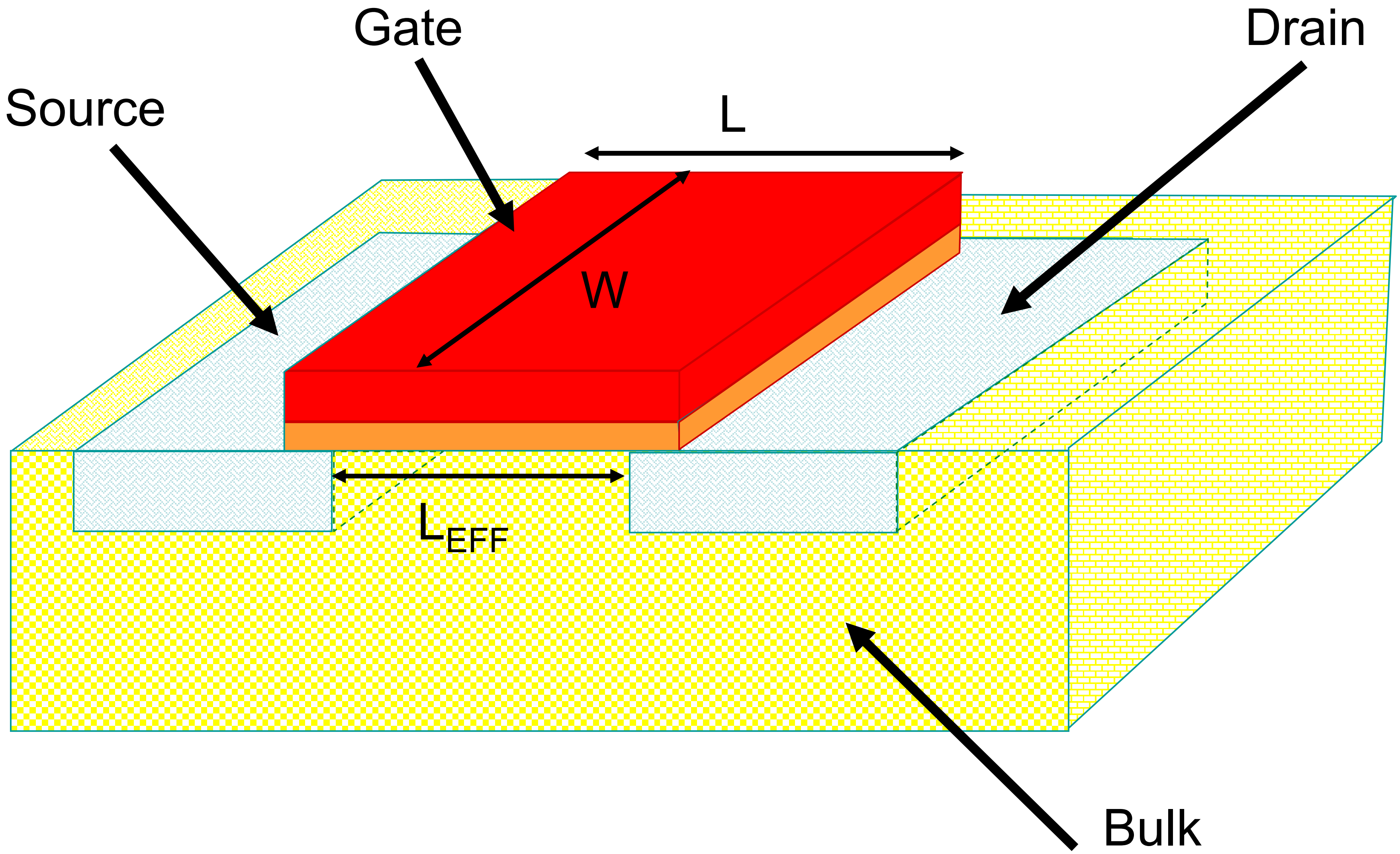
What is the minimum voltage at the drain of the MOS transistor that will cause the inversion layer at the interface between the drain and source to disappear? Assume the source is connected to ground and a dc voltage source of value V_{GSQ} is connected to the gate.



$$V_{DS}=V_{GSQ}-V_T$$

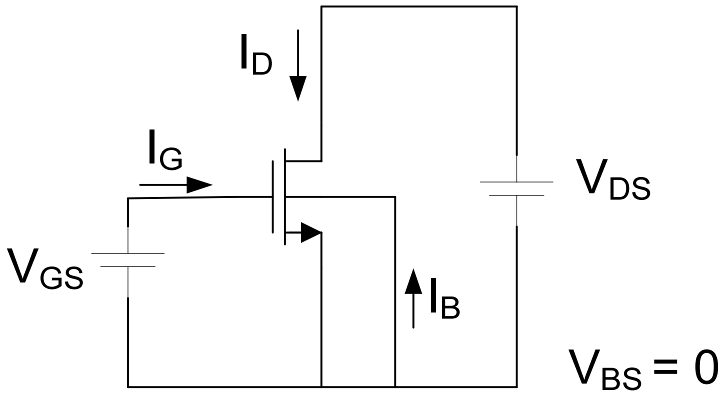
Review from Last Time

n-Channel MOSFET



Review from Last Time

Model Summary



$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

Cutoff

Triode

Sat.

- Model is nonlinear
- Model is piecewise & [illegible]

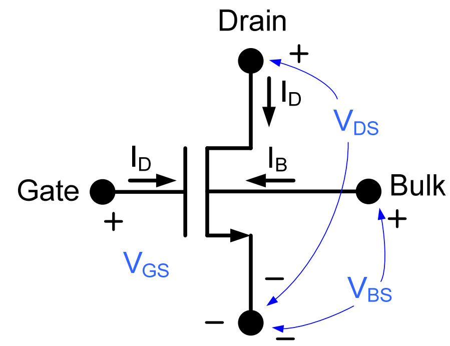
Note: This is the third model we have introduced for the MOSFET

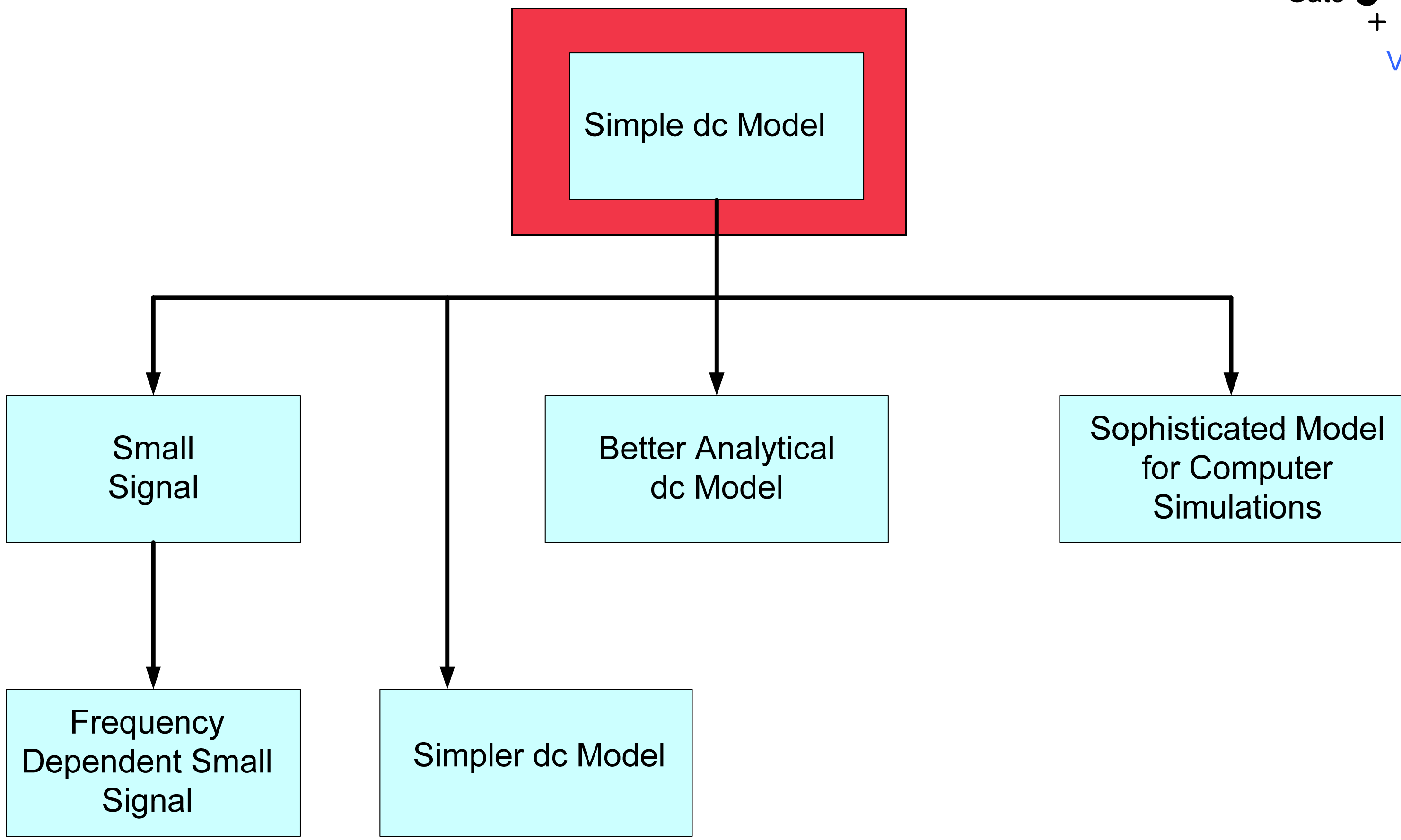
Review from Last Time

Modeling of the MOSFET

Goal: Obtain a mathematical relationship between the port variables of a device.

$$\left.\begin{aligned} I_D &= f_1(V_{GS}, V_{DS}, V_{BS}) \\ I_G &= f_2(V_{GS}, V_{DS}, V_{BS}) \\ I_B &= f_3(V_{GS}, V_{DS}, V_{BS}) \end{aligned}\right\}$$





Modeling of the MOSFET

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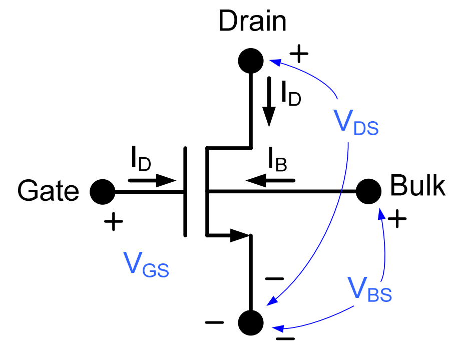
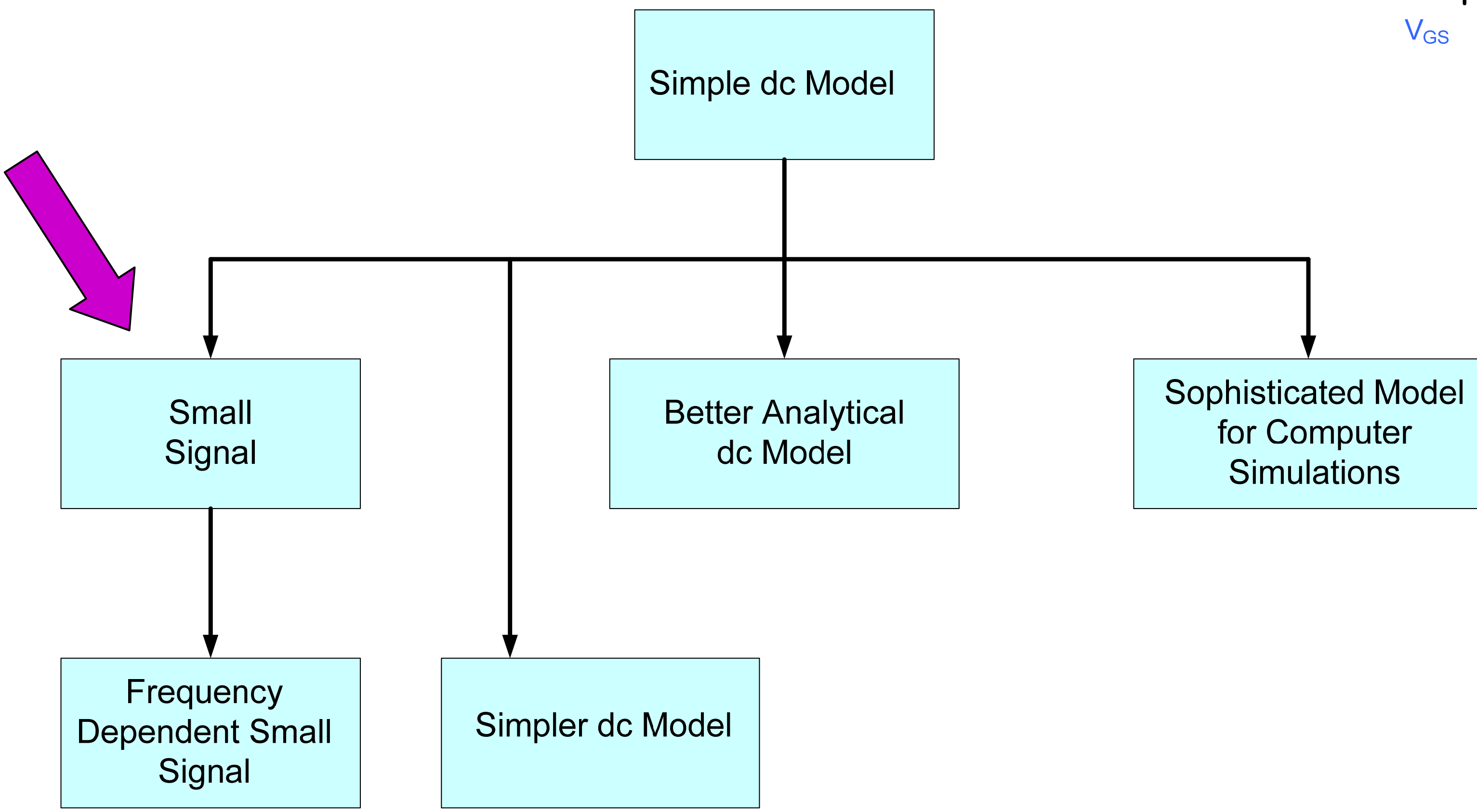




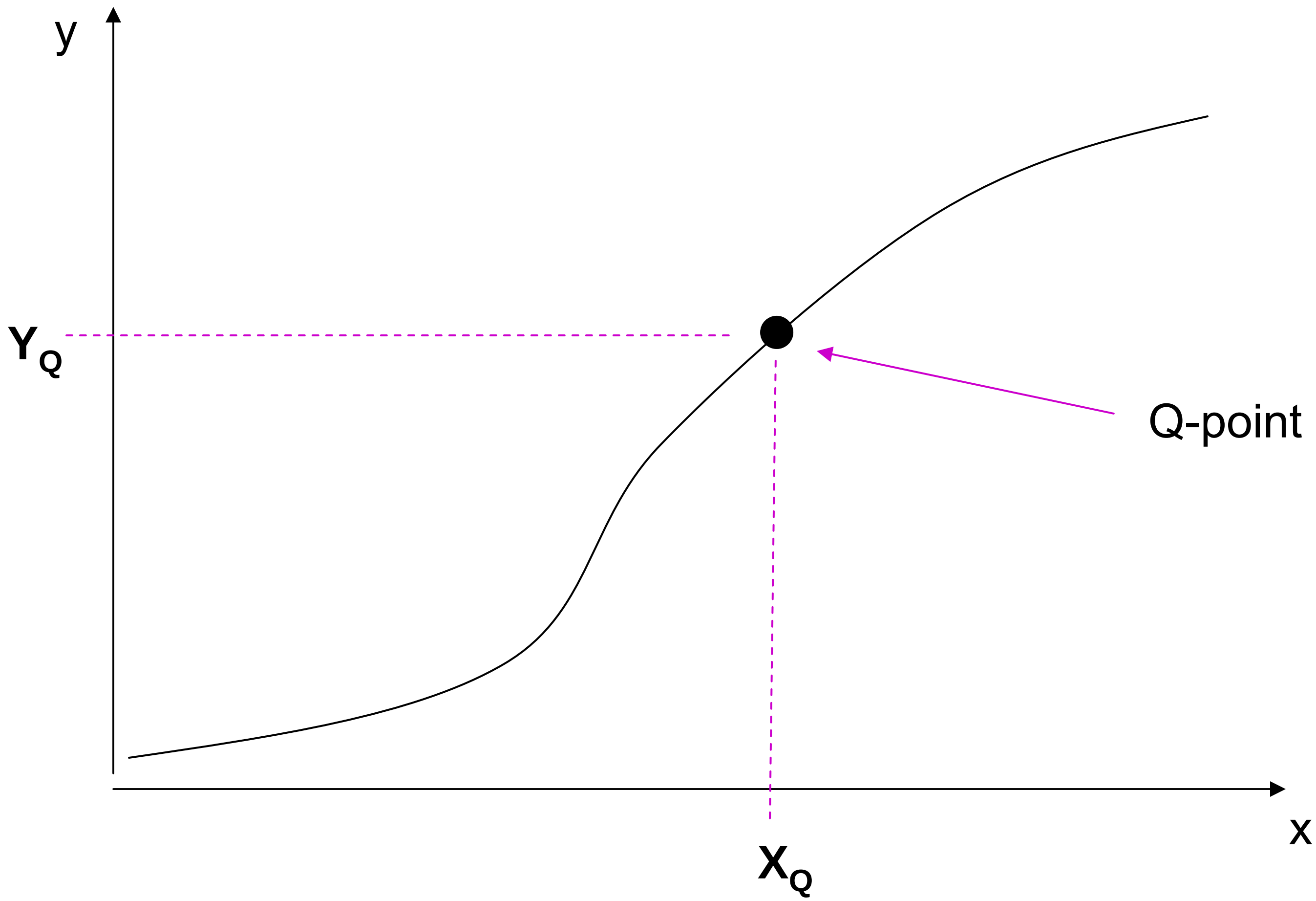
Small-Signal Model

Goal with small signal model is to predict performance of circuit or device in the vicinity of an operating point

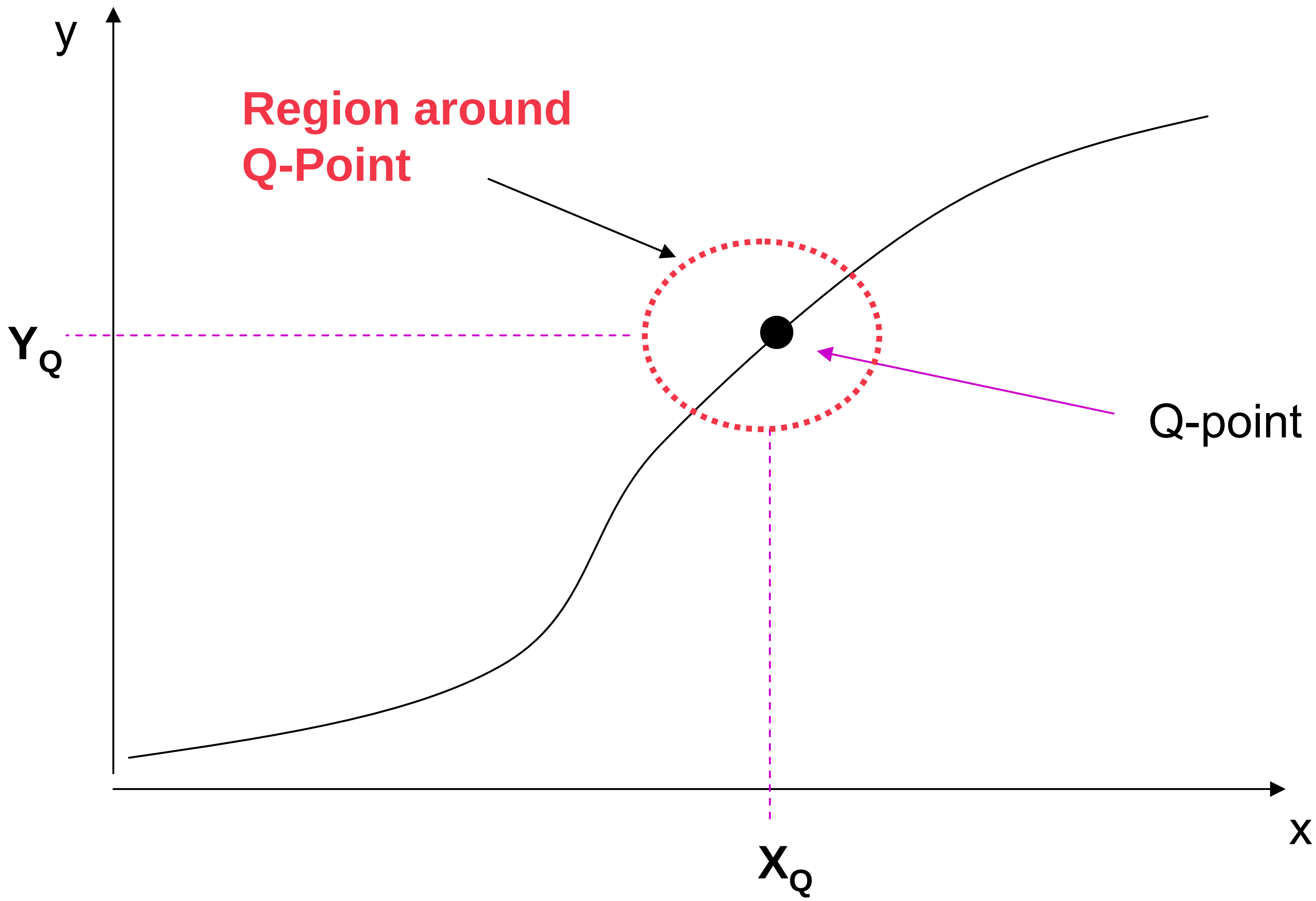
Operating point is often termed Q-point

Vicinity is a small region over which the device behaves linearly

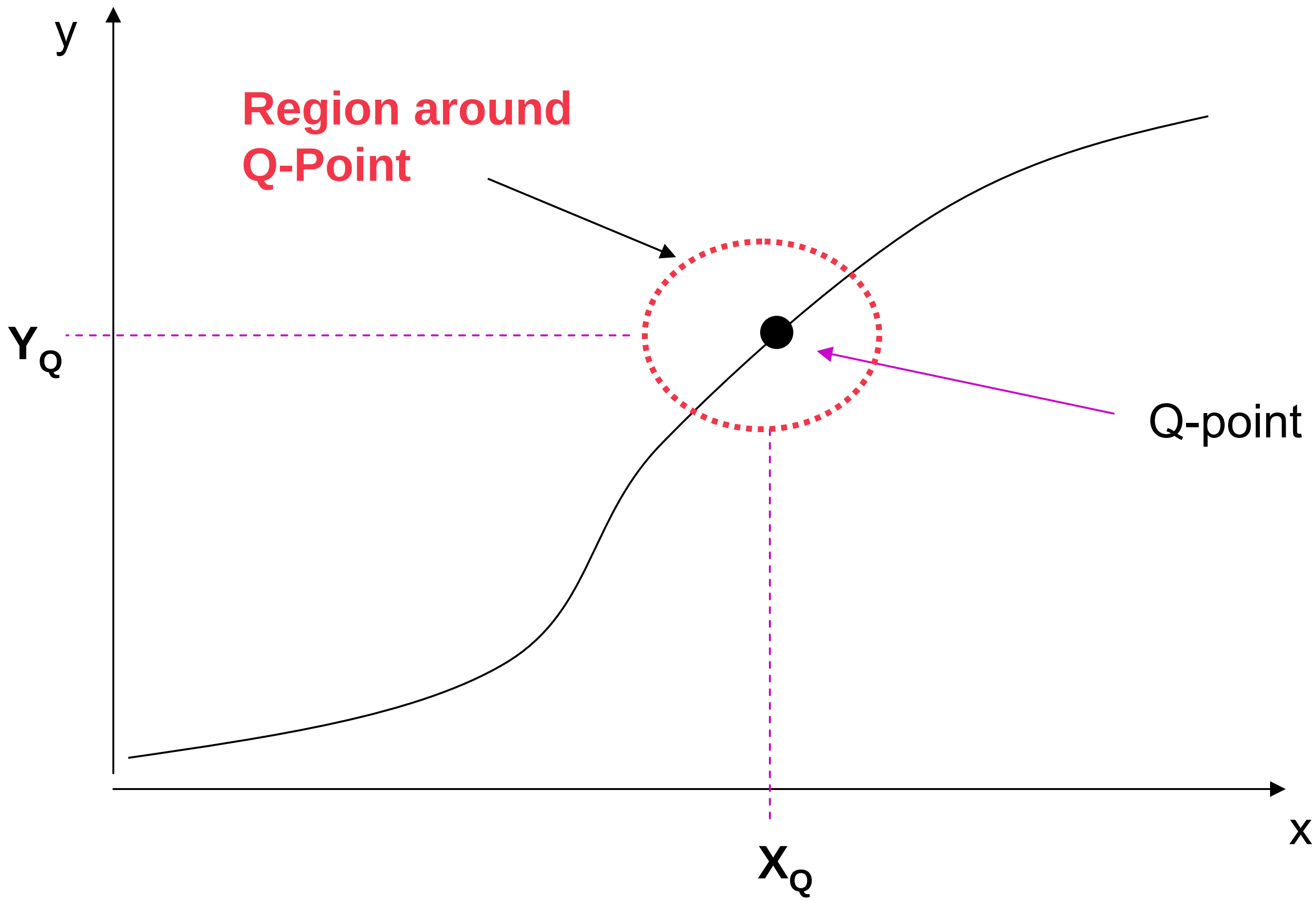
Small-Signal Model



Small-Signal Model

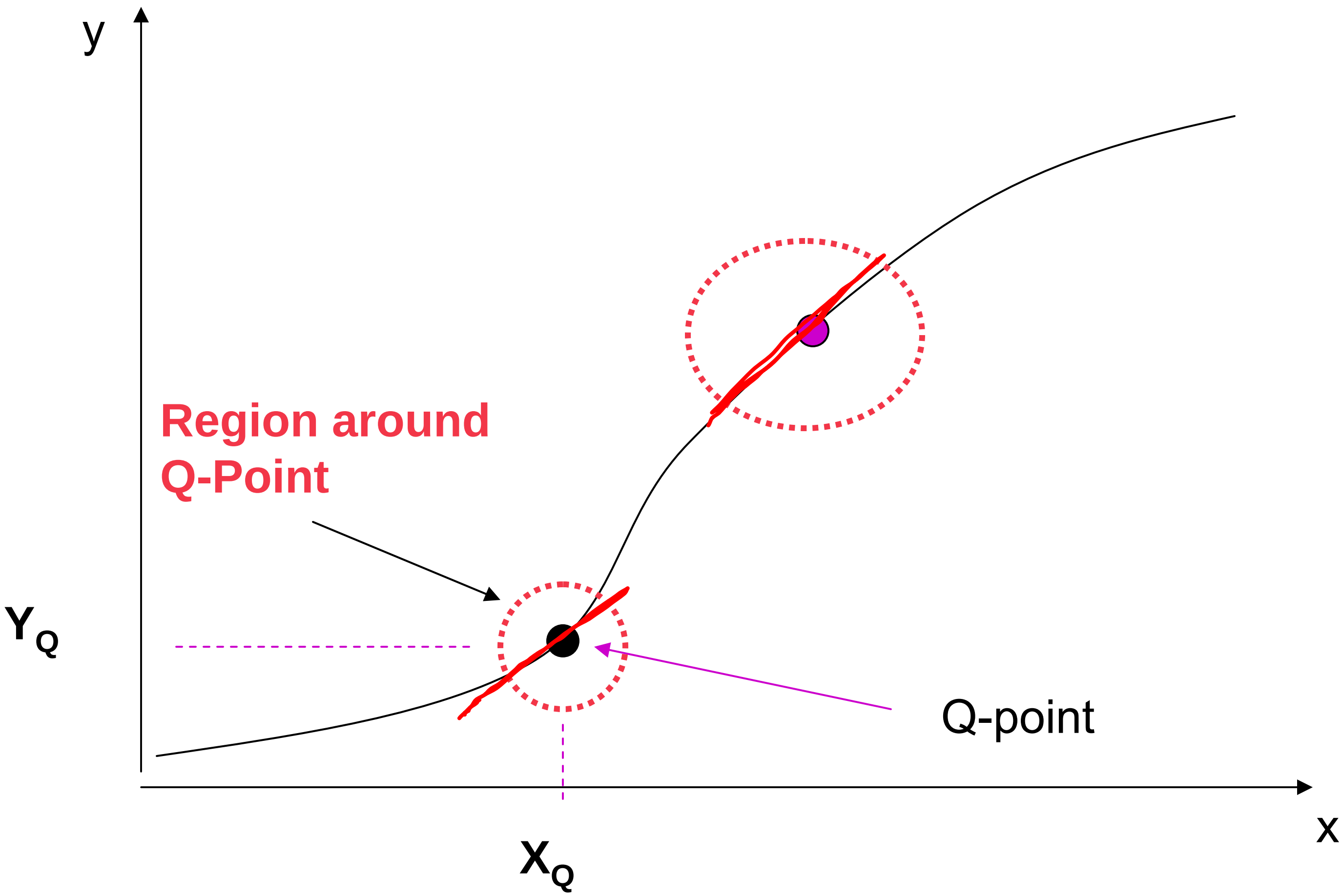


Small-Signal Model



Relationship is nearly linear in a small enough region around Q-point
Region of linearity is often quite large
Linear relationship may be different for different Q-points

Small-Signal Model



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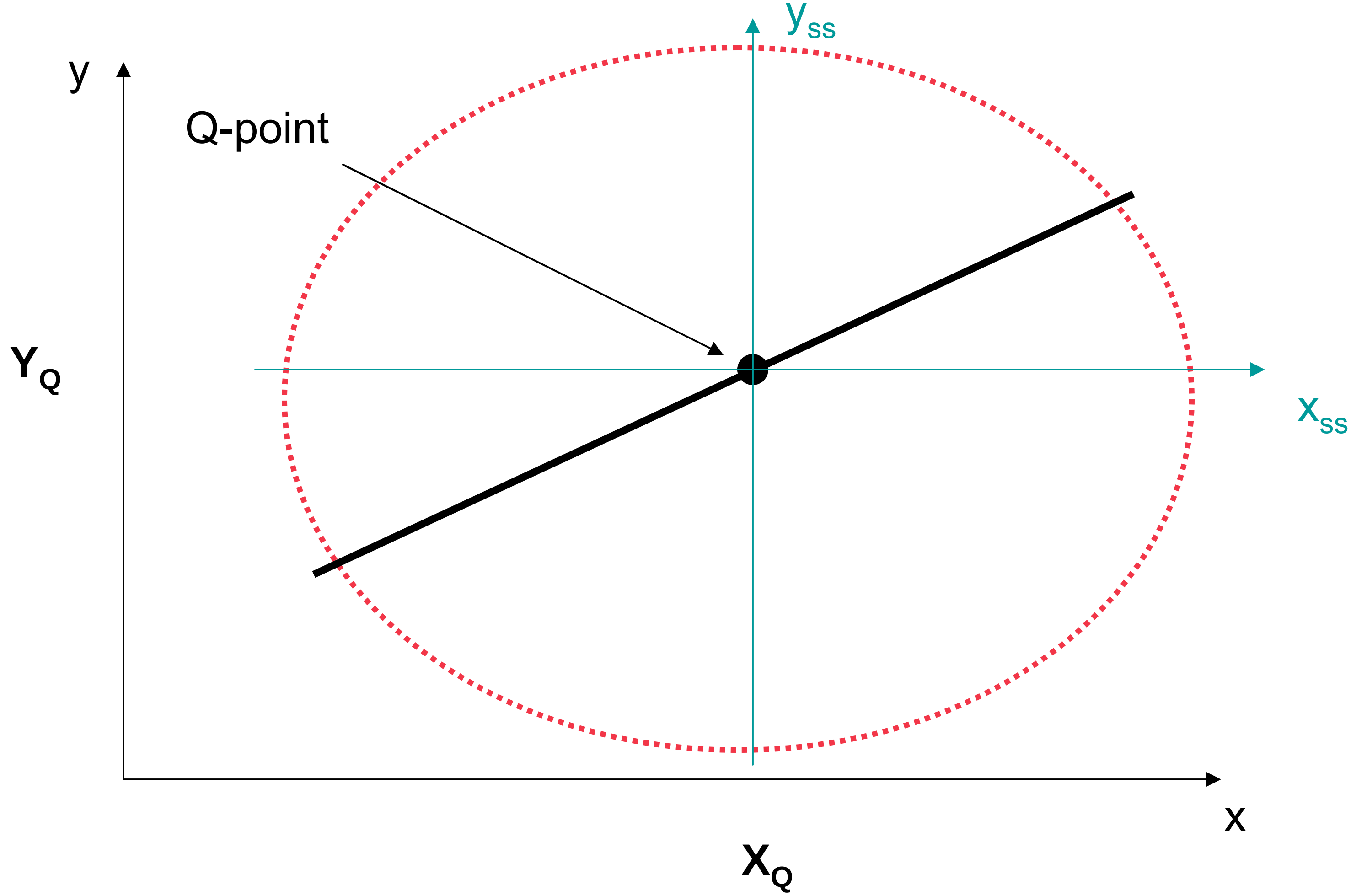
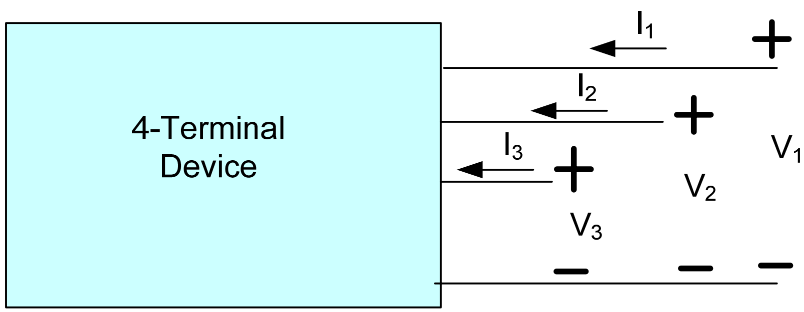
Small-Signal Model



Device Behaves Linearly in Neighborhood of Q-Point
Can be characterized in terms of a small-signal coordinate system

Small-Signal Model



$$\left.\begin{aligned} I_1 &= f_1(V_1, V_2, V_3) \\ I_2 &= f_2(V_1, V_2, V_3) \\ I_3 &= f_3(V_1, V_2, V_3) \end{aligned}\right\}$$

Define

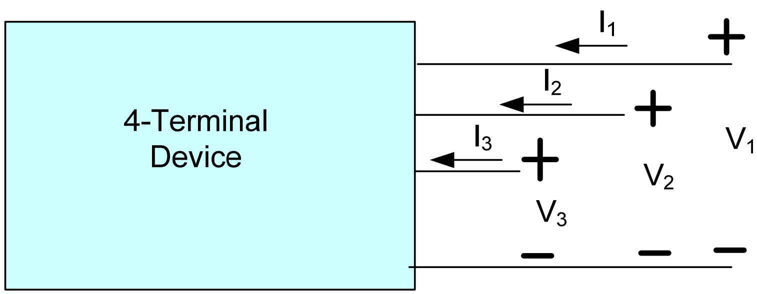
$$i_1 = I_1 - I_{1Q} \qquad v_1 = V_1 - V_{1Q}$$

$$i_2 = I_2 - I_{2Q} \qquad v_2 = V_2 - V_{2Q}$$

$$i_3 = I_3 - I_{3Q} \qquad v_3 = V_3 - V_{3Q}$$

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents

Small-Signal Model



$$i_1 = g_1(v_1, v_2, v_3)$$

$$i_2 = g_2(v_1, v_2, v_3)$$

$$i_1 = g_3(v_1, v_2, v_3)$$

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents

For small signals, this relationship should be linear

g_k is related to f_k

Recall for a function of one variable

$$y = f(x)$$

Taylor's Series Expansion about the point x_0

$$y = f(x) = f(x)\Big|_{x=x_0} + \frac{\partial f}{\partial x}\Big|_{x=x_0}(x - x_0) + \frac{\partial^2 f}{\partial x^2}\Big|_{x=x_0}\frac{1}{2!}(x - x_0)^2 + \ldots$$

If $x-x_0$ is small

$$y \cong f(x)\Big|_{x=x_0} + \frac{\partial f}{\partial x}\Big|_{x=x_0}(x - x_0)$$

$$y \cong y_0 + \frac{\partial f}{\partial x}\Big|_{x=x_0}(x - x_0)$$

Recall for a function of one variable

$$y = f(x)$$

If x-x_0 is small

$$y \cong y_0 + \left.\frac{\partial f}{\partial x}\right|_{x=x_0} (x - x_0)$$

$$y - y_0 = \left.\frac{\partial f}{\partial x}\right|_{x=x_0} (x - x_0)$$

If we define the small signal variables as

$$\boldsymbol{y} = y - y_0$$

$$\boldsymbol{x} = x - x_0$$

Recall for a function of one variable

$$y = f(x)$$

If $x-x_0$ is small

$$y - y_0 = \left.\frac{\partial f}{\partial x}\right|_{x=x_0} (x - x_0)$$

If we define the small signal variables as

$$\boldsymbol{y} = y - y_0$$

$$\boldsymbol{x} = x - x_0$$

Then

$$\boxed{\boldsymbol{y} = \left.\frac{\partial f}{\partial x}\right|_{x=x_0} \boldsymbol{x}}$$

This relationship is linear !

Consider now 3 functions each functions of 3 variables

$$\left.\begin{aligned} I_1 &= f_1(V_1, V_2, V_3) \\ I_2 &= f_2(V_1, V_2, V_3) \\ I_3 &= f_3(V_1, V_2, V_3) \end{aligned}\right\}$$

Consider now 3 functions each functions of 3 variables

$$\left.\begin{aligned} I_1 &= f_1(V_1, V_2, V_3) \\ I_2 &= f_2(V_1, V_2, V_3) \\ I_3 &= f_3(V_1, V_2, V_3) \end{aligned}\right\}$$

Define

$$\vec{V}_Q = \begin{bmatrix} V_{1Q} \\ V_{2Q} \\ V_{3Q} \end{bmatrix}$$

Consider now 3 functions each functions of 3 variables

$$\left.\begin{aligned} I_1 &= f_1(V_1, V_2, V_3) \\ I_2 &= f_2(V_1, V_2, V_3) \\ I_3 &= f_3(V_1, V_2, V_3) \end{aligned}\right\}$$

Define

$$\vec{V}_Q = \begin{bmatrix} V_{!Q} \\ V_{2Q} \\ V_{3Q} \end{bmatrix}$$

$$I_1 = f_1(V_1, V_2, V_3) \cong f_1(V_{1Q}, V_{2Q}, V_{3Q}) + \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1}\right|_{\vec{V}=\vec{V}_Q}(V_1 - V_{1Q}) + \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2}\right|_{\vec{V}=\vec{V}_Q}(V_2 - V_{2Q}) + \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3}\right|_{\vec{V}=\vec{V}_Q}(V_3 - V_{3Q})$$

$$I_1 - I_{1Q} = \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1}\right|_{\vec{V}=\vec{V}_Q}(V_1 - V_{1Q}) + \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2}\right|_{\vec{V}=\vec{V}_Q}(V_2 - V_{2Q}) + \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3}\right|_{\vec{V}=\vec{V}_Q}(V_3 - V_{3Q})$$

Consider now 3 functions each functions of 3 variables

$$I_1 - I_{1Q} = \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1}\bigg|_{\vec{V}=\vec{V}_Q}(V_1 - V_{1Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2}\bigg|_{\vec{V}=\vec{V}_Q}(V_2 - V_{2Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3}\bigg|_{\vec{V}=\vec{V}_Q}(V_3 - V_{3Q})$$

y_{11} $\quad$ y_{22} $\quad$ y_{23}

$$y_{11} = \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1}\bigg|_{\vec{V}=\vec{V}_Q}$$

$$y_{12} = \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2}\bigg|_{\vec{V}=\vec{V}_Q}$$

$$y_{13} = \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3}\bigg|_{\vec{V}=\vec{V}_Q}$$

$$i_1 = I_1 - I_{1Q}$$
$$i_2 = I_2 - I_{2Q}$$
$$i_3 = I_3 - I_{3Q}$$

$$v_1 = V_1 - V_{1Q}$$
$$v_2 = V_2 - V_{2Q}$$
$$v_3 = V_3 - V_{3Q}$$

Consider now 3 functions each functions of 3 variables

$$\mathbf{I_1 - I_{1Q}} = \left.\frac{\partial \mathbf{f_1(V_1,V_2,V_3)}}{\partial \mathbf{V_1}}\right|_{\vec{V}=\vec{V}_Q} \mathbf{(V_1 - V_{1Q})} + \left.\frac{\partial \mathbf{f_1(V_1,V_2,V_3)}}{\partial \mathbf{V_2}}\right|_{\vec{V}=\vec{V}_Q} \mathbf{(V_2 - V_{2Q})} + \left.\frac{\partial \mathbf{f_1(V_1,V_2,V_3)}}{\partial \mathbf{V_3}}\right|_{\vec{V}=\vec{V}_Q} \mathbf{(V_3 - V_{3Q})}$$

$$\boldsymbol{i}_1 = y_{11}\boldsymbol{v}_1 + y_{12}\boldsymbol{v}_2 + y_{13}\boldsymbol{v}_3$$

This is now a linear relationship between the small signal electrical variables

Consider now 3 functions each functions of 3 variables

$$i_1 = y_{11}v_1 + y_{12}v_2 + y_{13}v_3$$

Lets now extend this to I_2 and I_3

Define $\quad y_{ij} = \left.\dfrac{\partial f_i(V_1, V_2, V_3)}{\partial V_j}\right|_{\vec{V}=\vec{V}_Q}$

2-terminal (1)

3-terminal (4)

4-terminal (9)

$$i_1 = y_{11}v_1 + y_{12}v_2 + y_{13}v_3$$
$$i_2 = y_{21}v_1 + y_{22}v_2 + y_{23}v_3$$
$$i_3 = y_{31}v_1 + y_{32}v_2 + y_{33}v_3$$

$i_1 = g_1(v_1, v_2, v_3)$

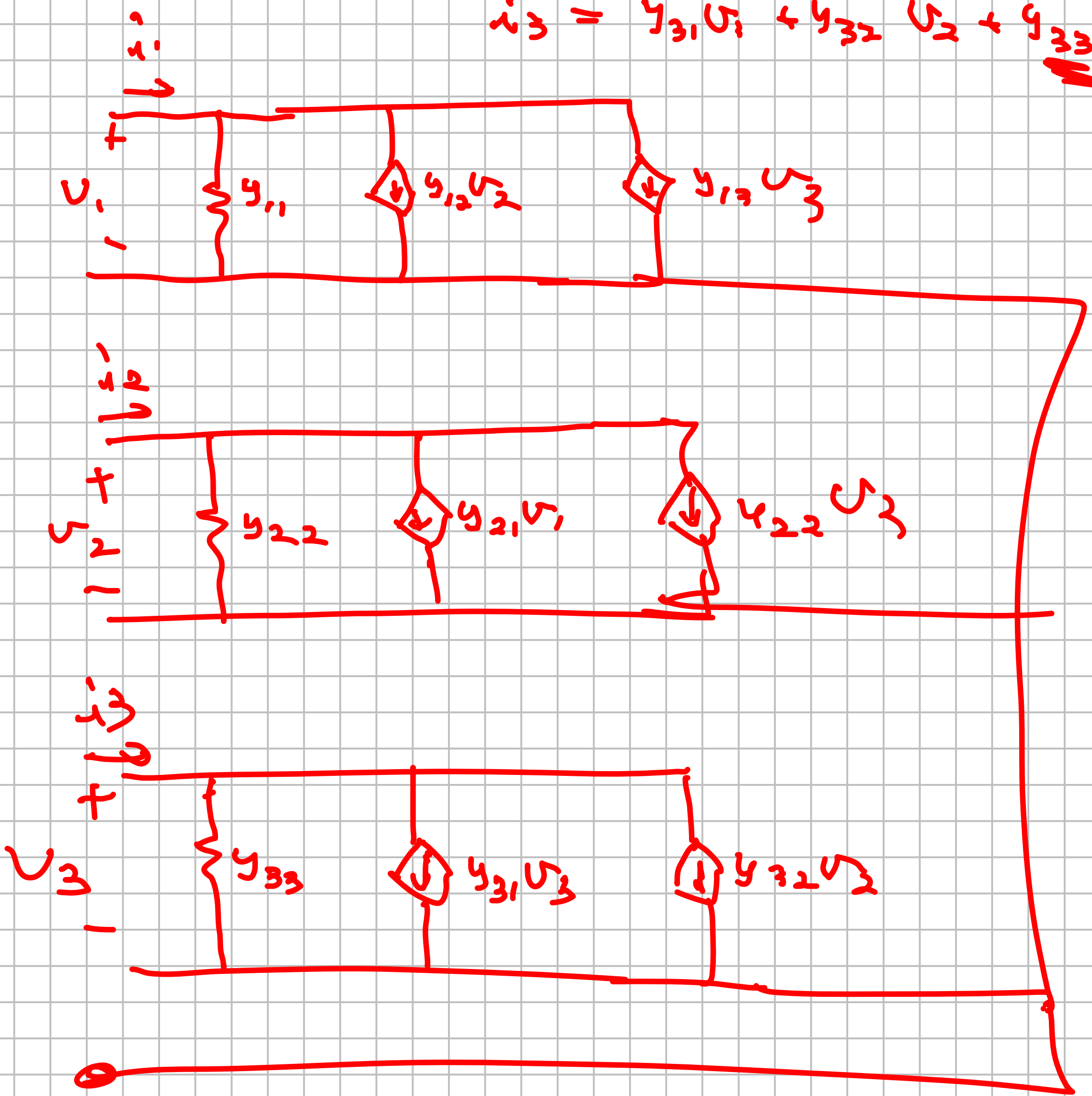
$i_2 = g_2(v_1, v_2, v_3)$

$i_3 = g_3(v_1, v_2, v_3)$

This is a small-signal model of a 4-terminal network and it is linear

An equivalent circuit for a 4-terminal network

$$i_1 = y_{11} v_1 + y_{12} v_2 + y_{13} v_3$$
$$i_2 = y_{21} v_1 + y_{22} v_2 + y_{23} v_3$$
$$i_3 = y_{31} v_1 + y_{32} v_2 + y_{33} v_3$$



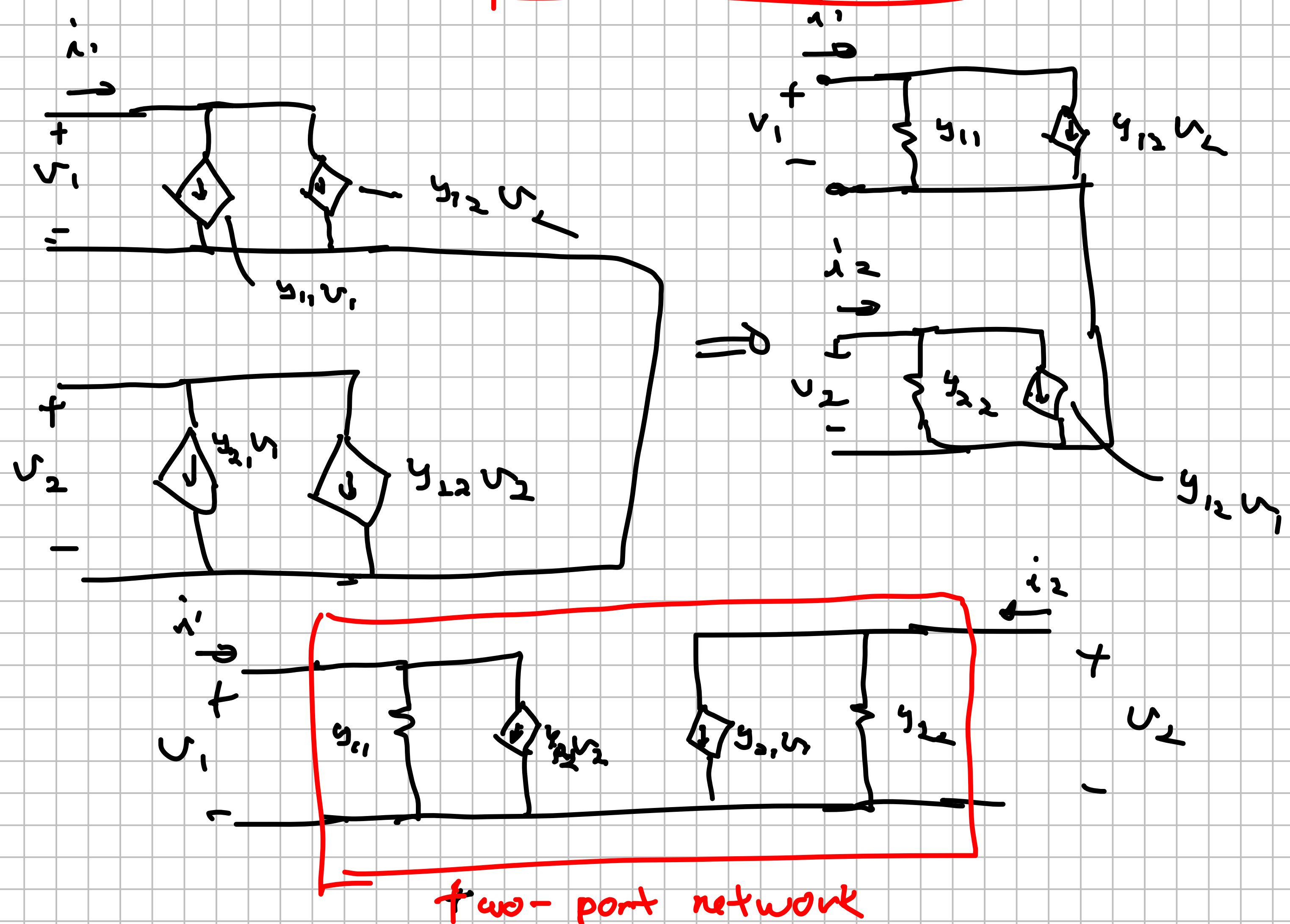
Note this equivalent circuit is not unique

There are actually many equivalent circuits

Small-signal model of 3-terminal device

3-terminal device

$$i_1 = y_{11} v_1 + y_{12} v_2$$
$$i_2 = y_{21} v_1 + y_{22} v_2$$



The 3-terminal device is often drawn as a "two-port" network

Equivalent circuits are not necessary

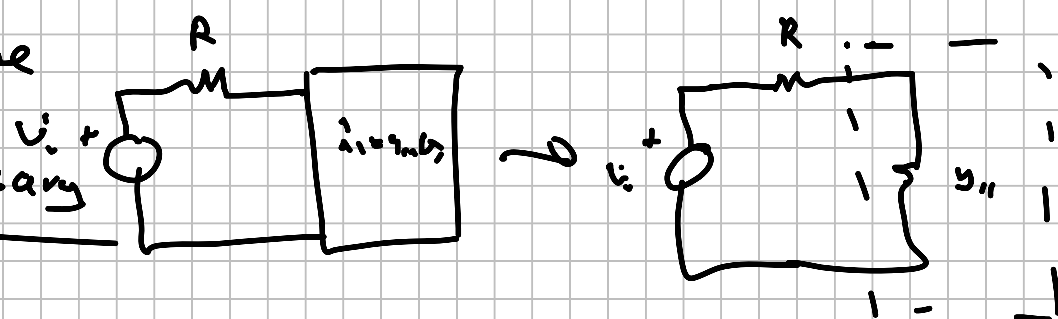
- Equivalent circuits are almost always given for small signal models

Engineers are adamant about having an "equiv circuit"

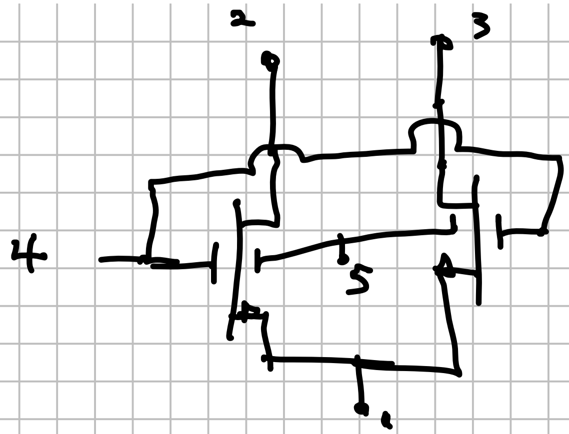
2 – terminal device

$$i_1 = y_{11} v_1$$

- equivalent circuits are readily obtainable
- not unique
- Not necessary



Example of a 5-terminal circuit



A small signal equivalent circuit

can be developed for this circuit too